Chapter 3

QNS Methodology

3.1 Particle Following Methodology

Understanding the physics and behavior of particles is essential to properly model particle motion in turbulent flows. A key ingredient in understanding such behavior is the knowledge of particle and fluid statistics from the reference frame of the particle or the particle-Lagrangian (p-L) reference frame. The Tufts University Fluid Turbulence Lab (TUFTL) developed a system to emulate virtual particles within a real turbulent flow allowing fluid velocity measurements to be made from within the p-L reference frame.

Water within a water channel in essence solves the Navier-Stokes equations using real physics, while particle motion is governed by the particle transport equation. A simplified form (see section 1.4) of the equation

$$\frac{\partial \vec{V}_p}{\partial t} = \frac{1}{\tau_p} (\vec{U}_f - \vec{V}_p) - \vec{g},$$

(3.1)

where $\vec{U}_f$ is the fluid velocity and $\vec{V}_p$ is the particle velocity, is solved to determine particle accelerations which is simulated using a traverse system which follows the path of the emulated particle. A laser Doppler velocimetry probe (LDV) attached to the traverse takes measurements from what is now the p-L reference frame. Additional devices, such as a camera can also be attached to take data as well.
The system relies on a control loop using a discretized version of equation (3.1) running continuously on a DSP card to take feedback from the LDV system and update the traverse acceleration to match the virtual particle. The discretized form of (3.1)

\[
\frac{V_p^{n+1} - V_p^n}{\Delta t} = \frac{1}{\tau_p} (V_p - U_f)^n - g
\]  

(3.2)

has the term

\[(V_p - U_f)^n\]  

(3.3)

which corresponds to the velocity measured by the LDV probe. Equation (3.2) can be solved yielding

\[
V_p^{n+1} = \frac{\Delta t}{\tau_p} U_{measured}^n + V_p^n - \Delta tg
\]  

(3.4)

which is the velocity sent to the traverse. This equation is solved continuously within the control loop in order to emulate the virtual particle with the traverse. The emulated particle’s location corresponds to the measuring volume of the LDV probe from which the value of term in equation (3.3) is determined.
The previous version of the control code used by Ainley [1] would poll the velocimetry system every 5ms to check for a new data point. The particle velocity from the transport equation would then be updated with this new value. The $\Delta t$ in equation (3.4) corresponded to the time between available data points. If the effective data rate from the velocimetry system exceeded one point every 5ms, some data would be discarded, limiting the maximum traverse update rate to 5ms. Computational overhead further limited the maximum traverse update rate to once every 10 ms. Gaps in the data rate larger than this would slow the overall update rate and the traverse would maintain a constant velocity between these gaps in velocimetry data.

Data for this thesis used a new particle advancement scheme that maintained a constant traverse acceleration rather than velocity. The new system also utilized faster velocimetry data rates allowing the control system to be updated faster than
the previous maximum of 100 Hz. Figure (3.3) is a flowchart of the new particle advancement algorithm utilized. This algorithm was computed continuously with the acceleration rate being updated each time a new data point was available from the LDV system. The velocity commands sent to the traverse servos were updated once every time the algorithm was executed. The code, which ran on a DSP card and was unaffected by other computer processes, produced an effective \( \Delta t \) on the order of microseconds.

![QNS Algorithm Flowchart](image)

Figure 3.3: QNS algorithm flowchart.

To properly follow the flow the data rate must remain high enough to detect all changes within the flow and have the traverse react to these. This requires that the data rate from the LDV system be high enough to detect all fluid fluctuations.
This implies that the $\Delta t$ must be smaller than the Kolmogorov time scale of the water flow. The average data rates used in the experiment ranged between 175-275 Hz corresponding to a $\Delta t$ between 3.6 and 5.3 ms compared to the estimated Kolmogoroff scales between 60 and 200 ms for the Reynolds numbers tested. This order of magnitude difference provides ample leeway for emulating the particle. The traverse response rate was also found to range between 2 ms and 15 ms depending upon the axis and tuning parameters used with different masses attached to the traverse.

The following figures qualitatively demonstrate the performance of the system. As figure (3.4) shows, large simulated particles hardly react to the turbulence and completely ignore the higher frequency fluctuations due to their inertia. This is akin to a bowling ball moving through the air - it totally ignores the small scale structures of the turbulence. As the particle time constant decreases, the particle begins to react to more and more of the turbulence as further figures show.

![Figure 3.4: Particle Velocity vs. Fluid Velocity ($St_k = 44$)](image-url)
Figure 3.5: Particle Velocity vs. Fluid Velocity ($St_k = 18$)

Figure 3.6: Particle Velocity vs. Fluid Velocity ($St_k = 12$)
The size of the particles that can be emulated is restricted due to an inherent limitation of the LDV system. The velocimetry system requires that seed particles within the flow pass through the measuring volume to produce a data rate. As the time constant of the virtual particle is decreased and the particle begins to follow the flow more closely, fewer seed particles pass through the measuring volume and causing the data rate from the system to drop to the point that it is no longer sufficient to capture the nuances of the flow.

3.2 Sources of Error

Errors introduced from uncertainties in the LDV system and errors in positioning the traverse can lead to overall system errors that effect the results. Both Ainley [1] and McAndrew [14] discussed these experimental errors. A brief overview and slight expansion is given below.
3.2.1 LDV Uncertainty

The frequency registered by the LDV system was converted to velocity measurements and digitized using an 11-bit A/D converter in the IFA 750 receiver. Therefore, there are 2048 possible measurement values between the high and low pass filters set within the LDV system. The uncertainty produced by the finite resolution of the A/D is given by

\[ u = \frac{F_{LP} - F_{HP}}{2^{11}} \]

where \( F_p \) is the frequency produced by the seed particle passing through the measuring volume. The velocities measured by the LDV probe in this thesis used a band pass filter range between 30-300 Hz and a shift frequency of 100 kHz with corresponding uncertainties less than 1%.

Ainley also investigated errors introduced by velocity bias, incomplete signal bias, gradient broadening, and ensemble averaging uncertainty, concluding that the overall uncertainty never exceeded 2%.

3.2.2 Traverse Uncertainty

The traverse system had to be calibrated so a given command voltage corresponds to a particular traverse velocity. Each axis was calibrated by blocking each end of the narrow channel producing stationary water which has seed particles suspended within it. The traverse with LDV system was commanded to move while the LDV system recorded fluid velocities. The commanded velocities could then be compared against the velocities measured by the LDV probe which should correspond to an absolute reference frame since the flow was stationary. Using this method, the commanded to true velocities could be calibrated to within 0.00001 m/s or 0.03% of the mean velocity at \( Re = 6600 \). Using a similar method, both Ainley and McAndrew studied the response of the traverse to guarantee that the traverse moved at the commanded velocity within the limits of the response time. Using a real particle path, deviations
from expected traverse velocities were within 1% for both axes. Coppen [3] further expanded on this using a quadrature encoder and confirmed that the traverse was successfully following the prescribed particle path.