ABSTRACT
The goal of this work is to measure turbulence statistics in the reference frame of a moving particle. Since particles have inertia and weight, their paths are neither the classical Lagrangian one of a fluid point nor the Eulerian one of a stationary point. Instead, their path is dictated by the instantaneous fluid drag and weight, hence they follow a particle-Lagrangian reference frame. Since the particle responds to the turbulence measured in this reference frame, one must be able to estimate the turbulence in this reference frame in order to model the particle behavior accurately. In this investigation, we track an imaginary particle through a fully-developed channel flow using a rapidly moving laser Doppler velocimeter (LDV) system. The path of the imaginary particle is determined by measuring an instantaneous fluid velocity and then using a simplified Stokes transport equation to determine the appropriate acceleration at which the LDV probe is to move. Thus, the statistics measured by the LDV would be those seen by a particle moving in the turbulence field. The experimental work is in progress and the results shown will be of preliminary runs at Re of 6000. These findings will be compared to the results of a similar direct numerical simulation of similar Reynolds number as well to Eulerian statistics from the same flow field.

INTRODUCTION
Particle-laden channel flow is a common occurrence in both nature and industry, yet it is not well understood, due mainly to the lack of experimental work in the field. For example, coal pulverization, dust storms, atmospheric pollution transport, and the spray processing of materials all are difficult to predict. Improved models will allow better optimization of these phenomena. In most of these situations, the particles occupy only small a fraction of the total volume. As an example, solid particles making up one half the mass of a gas-solid mixture might occupy less than one-tenth of one percent of the total volume. This makes inter-particle collisions be of little importance, implying that the particle dispersion is a direct result of the fluid turbulence.

Although there have been extensive modeling efforts in the area of particle dispersion as well as particle/turbulence interaction, most efforts have been limited by the difficulties in predicting the
turbulence as experienced by the particle. Gravitational and inertial forces will cause the particle to deviate from the classical Lagrangian reference frame of the fluid point. Thus, the turbulence the particle responds to may different from the turbulence measured in either the classical Lagrangian or Eulerian frames. For instance, a particle rapidly propelled through a frozen eddy will “see” a higher frequency than a corresponding slower particle going through the same eddy. Because the particle reacts to the turbulence in its reference frame, one must be able to estimate the turbulence in the particle-Lagrangian reference frame to accurately predict the particle behavior. For instance, when modeling the particle dispersion, modelers have either used a simple Schmidt number model or a model based on Taylor’s work in the dispersion of a fluid point (Taylor 1921). The Schmidt number model fails to reflect much of the underlying physics since diffusion is not a true dispersion process due to gravitational and inertial forces. Taylor’s result describes the mean square dispersion of a fluid point (Y) as a function of the fluid autocorrelation:

\[
\overline{Y^2(t)} = 2v'^2 \int_0^t \int_0^t R(\tau)d\tau ,
\]

(1)

where \(R\) is the fluid autocorrelation along the classical-Lagrangian reference frame (Hinze 1975). Thus one must estimate the particle autocorrelation from the fluid autocorrelation. This must be the fluid autocorrelation in the particle-Lagrangian reference frame. Likewise, when modeling the particle/turbulence interactions, one must be able to estimate the particle-fluid velocity correlation - which again is based on the fluid autocorrelation in the particle-Lagrangian reference frame. Kulick et al found that the attenuation of turbulence is strongly dependent on the particle time constant. In order to understand or model this effect, we must be able to calculate the correlation between the fluid and particle velocity fluctuations. This requires knowledge of the particle-Lagrangian autocorrelation technique.

In order to measure this autocorrelation, a system must be developed which can follow a moving particle while simultaneously measuring the fluid velocity surrounding the particle. Alternatively, particle-Lagrangian statistics can be calculated from direct numerical simulations (e.g. Squires and Eaton 1991). Such simulations, however, are limited to low Reynolds numbers and very simple flow geometry. The objective of this research is to make actual measurements in this particle-Lagrangian reference frame. The idea is to have a flow in which the velocity is being measured by a laser-Doppler velocimeter (LDV). Using the LDV signal, the computer calculates the acceleration in two dimensions of an imaginary particle and then accelerates the LDV at these calculated rates. Therefore, the turbulence statistics measured by the LDV are those in the particle-Lagrangian reference frame. Because there is no particle in the flow, this technique only works for particles which do not affect the surrounding flow (small Reynolds number). Further, the measured turbulence statistics are those in the vicinity of the particle, yet not directly affected by the particle’s presence. That is, we cannot measure the small accelerations of the fluid around the particle itself as there is no particle in the flow. With these two limitations, however, we can, with an adequate LDV signal rate, measure the autocorrelation of both the particle and the fluid in the particle-Lagrangian reference frame. The one other limitation of the system is that the imaginary particle is constrained to move only in two dimensions - since the LDV is traversed only in two dimensions. This limitation can be assessed through direct numerical simulation and is addressed further in the paper.

The path of the imaginary particle, and therefore the LDV, is determined by the particle-transport equation. The full transport equation of Maxey and Riley (1983) is:
\[
\begin{align*}
\frac{\pi d_p^3 \rho_p}{6} \frac{dV_i}{dt} &= \frac{\pi d_p^3 (\rho_p - \rho_f) g_i}{I} - \\
3\pi \mu_p \left( V_i - U_i - \frac{1}{24} d_p^2 \nabla^2 U_i \right) &+ \frac{\pi d_p^3 \rho_f}{6} \frac{DU_i}{Dt} - \\
\frac{\pi d_f^3}{12} \frac{dV_i}{dt} - \frac{DU_i}{Dt} - \frac{1}{40} d_p^2 \nabla^2 U_i &- \\
\frac{3}{2} \pi d^2 \mu f \int_0^t d\tau \left[ \frac{d}{d\tau} \left( V_i - U_i - \frac{1}{24} d_p^2 \nabla^2 U_i \right) \right] &+ \\
\pi \mu_v t &- \tau \left( U_i - V_i \right) + g_i .
\end{align*}
\]

where
- \( d_p \) = particle diameter
- \( \rho_p \) = particle density
- \( \rho_f \) = fluid density
- \( g_i \) = gravity
- \( \mu \) = dynamic viscosity
- \( V_i \) = particle velocity
- \( U_i \) = fluid velocity.

Term I on the left-hand side of the equation represents the change in the inertia experienced by the particle. Term II is the gravitational force on the particle. Term III depicts the Stokes force due viscous drag. Term IV represents the force due to the pressure gradient of the fluid. Term V characterizes the force needed to accelerate the fluid into the space left by a moving particle. Finally, term VI is the Basset history force which accounts for the past history of the particle.

Although one could theoretically use the full equation, we reduced the equation to the case of particle densities being orders of magnitude greater than the density of the carrier fluid. Equation 2 then becomes

\[
\frac{dV_i}{dt} = \frac{1}{\tau_p} \left( U_i - V_i \right) + g_i ,
\]

where

\[
\tau_p = \frac{\rho_p d_p^2}{18 \mu}
\]

for particles with small \( \text{Re}_p \). This is the equation used by the computer to accelerate the LDV. The user can choose both gravity and the particle time constant. Thus, in essence, we are doing a numerical simulation with the water solving the instantaneous Navier-Stokes equations.

**EXPERIMENTAL SETUP**

*Apparatus*

The experimental setup, shown in figure 1, consists of a water-flow tank, 0.15 m diameter piping, and a two-dimensional traverse system. The flow tank consists of three sections: the entrance region, channel, and collection region, respectively, with the channel consisting of a development section and a test section. The channel section is 4.87 m long (240h) and has a cross section 0.5 m in height and 2 cm half width (h). Water flow is in the counterclockwise direction and is driven by a propeller.

The traverse system consists of a two-dimensional table (streamwise and vertical directions). The streamwise traverse is a roller belt driven Parker-Daedal positioning table and has a travel of 2.4 m, allowing for ample ramping to and down from its desired velocity. The vertical table is driven by a 0.005 m pitch ballscrew with a 1 m of travel, easily covering the entire height of the channel. We are currently characterizing the acceleration characteristics of the traverse.
Controls
The LDV velocity is controlled by the control loop in a Macintosh PowerPC 8100/80AV. Flow velocity data is acquired with a two-dimensional Bragg-shifted backscatter TSI LDV probe and a TSI IFA 750. The LDV measuring volume is 34µm in diameter and 180 µm long. Based on the simulations, the Kolmogorov length scales should be on the order of 200-300 µm. Further, we are currently updating the control loop at 12 Hz. Again, based on the simulations, the Kolmogorov time scales should be around 100 msec. We should be, therefore, accurately following even the smallest scales in the flow. We are currently working on increasing the speed of the control loop so that we can go to higher Reynolds number channel flows.

The actual control code calculates the LDV acceleration based on the latest LDV measurement. Since we had coincidence data rates always in excess of 200 Hz, this meant a new acceleration with each iteration. The code uses this acceleration to compute a 256 point voltage ramp for each servo motor. These motors are controlled with Parker - Daedal BL Series PID controllers. Figure 2 shows a schematic of the control code.
Our main concern is how closely is the traverse responding to the voltages sent out by the control code. We are currently working on measuring this by repeating each run with the LDV focused on a stationary screen. Thus, the effective screen velocity should be the velocity of the probe. This can be compared to the desired velocity to measure the response of the system. We can also compare to the velocity measured in the PID controllers, based on the rotary optical encoders which are mounted on the servo motors.

RESULTS
Channel Characterization
Figure 3 shows the velocity measurements across the height of the channel. Figure 3a is a plot of the streamwise velocity normalized by the centerline velocity as a function of vertical position, normalized by channel half-width. The two lines are profiles at the entrance and exit to the test section and deviate less than 1% - implying that the flow is fully developed. Although one can see two percent variation across the channel, the probe moves in the region of $10 < \frac{y}{h} < 15$, where the velocity variation is less than one percent. We are currently refining the channel to reduce this variation. The velocities in the vertical direction show no mean velocity, as one would expect.

Particle (Traverse) Velocity
The next set of plots reflect the velocity of the traverse and therefore of the imaginary particle with a time constant of 1 second and no gravity. Figure 4 is the streamwise velocity of the particle relative velocity ($U_f - V_p$) as a function of time. The particle velocities are normalized by the mean Eulerian flow and the time axis is normalized by the particle time constant. On average, the traverse behaves as one would expect; accelerating to the fluid velocity and then continuing at the flow velocity (gravity has been neglected). The ideal case on the plot is that of no fluid turbulence. Equation 3 then can be analytically solved as a decaying exponential. One can see that the runs are in very good agreement with that ideal case on average. The case of the instantaneous run, however, shows the effect of the fluid turbulence on the path of the particle. As one would expect, the vertical velocity (figure 5) of the traverse is zero on average. The instantaneous runs, again, show the effects of the turbulence fluctuations. All averaging was done by separating the time axis into 250 millisecond bins and then ensemble averaging across 33 runs.
Measurements of Particle Fluctuations

With the particle velocity measurements attained, we could measure the particle (or traverse) rms. velocity (figure 6). Again, these plots show ensemble averages over thirty-three runs. Although one can see evidence of the small number of runs, the average rms. measurement in the particle-Lagrangian reference frame is within two percent of the Eulerian measurement in the streamwise direction. This implies that the particle was accurately following almost all of the scales in the flow. Again, this was done for a particle time constant of 1 sec. We have not yet measured a Kolmogorov time scale in our flow, but it should be on this order (Stokes numbers order unity).

Contrary to the horizontal case, the particle fluctuations in the vertical direction (figure 7) are, on average, 20% lower than the Eulerian measurements. This could be a result of our low number of ensemble averages or a result of the Stokes number of our flow. Particles (the traverse) with Stokes’ numbers on the order of one will not follow the higher frequency fluctuations of the flow and therefore will read a lower rms. velocity. Since the vertical velocity tends to have more energy in the higher frequency, it would be the first to be affected. At this stage of the experiment, however, we do not have enough data to substantiate this claim.

Direct Numerical Simulation

The most worrisome aspect of the present experiment is that the imaginary particle is constrained to move within a two-dimensional plane - that is we do not traverse the LDV along the width of the channel. Therefore, it cannot represent the effects of the out of plane motions which might affect the statistics. There does not appear to be any analysis that will allow us to convert the two-dimensional measurements to those obtained in full three-dimensional motion.

In order to assess the effects of this constraint, we have conducted some preliminary direct numerical simulations of the particle motions in a three-dimensional channel flow. The DNS code is a spectral code with uniform gridding in the X and Z directions. The grid is a 128 x 128 x 128 grid and...
actively tracks $72^3$ particles. Details of the simulation techniques are well documented (Rouson and Eaton 1994). In these simulations, we compared particles moving in three dimensions to particle constrained to the two-dimensional plane of the experiment. These simulations were conducted at a bulk Reynolds number of 1550, and we are currently looking at the case of higher Reynolds numbers. Simulations were done for two different particle Stokes numbers, $St=0.4$ and $St=31$ - where the fluid time scale is taken to be Kolmogorov time scale at the channel centerline. Figure 8 shows the particle-Lagrangian autocorrelation coefficient for all three velocity components at $St=0.4$. The time axis is normalized by wall units and the vertical axis is the normalized autocorrelation defined as:

$$ R_{\alpha\alpha} = \frac{\{U_\alpha(t_0 + \tau)U_\alpha(t_0)\}}{\{U_\alpha(t_0)U_\alpha(t_0)\}}. $$

(5)

The solid line represents the autocorrelation coefficients for the particles with three degrees of motion, and the symbols are for those constrained to the plane. Agreement is good for the streamwise and wall normal velocity components, but the deviation in the spanwise direction is large. The reason for this is unclear and further computations are underway to evaluate this. Figure 9 shows the same plot for the higher time constant particle; that is the less responsive particle. Here the constrained particle qualitatively shows the correct features. As one might expect, however, the constrained particles tend to stay correlated over longer times than the unconstrained particles. We hope to use further simulations to find an appropriate scaling to estimate the autocorrelation coefficients for the unconstrained particle from the measured autocorrelation for the constrained particle.

**CONCLUSION**

In conclusion, this research presents a novel technique of measuring in the particle-Lagrangian reference frame. Although it is still in its early stages, the preliminary data follows the expected trends. There are, however, two limitations inherent to the work. First, we have confined our “particle” to two dimensions. Second, the local disturbance of the flow due to the physical presence of the particle cannot be assessed since there is no physical particle in the flow.

Using DNS results, we can estimate the effects of the two-dimensional confinement. Preliminary work shows these effects to be small. Since we are simulating small $Re_p$ particles, the effects of local disturbances we measure should
be accurate and have a substantial effect on improving the current models.

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REFERENCES


