A Technique for Fluid Velocity Measurements in the Particle-Lagrangian Reference Frame

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Abstract

The goal of this paper is to present a new technique for measuring fluid velocities in the reference frame of a solid particle in air and to compare data obtained using this technique to existing data. The particles are assumed to have much greater density than the surrounding fluid and the flow around the particle is Stokesian. The modeling of the particles requires that the fluid turbulence statistics are measured in the reference frame of the moving particle (particle-Lagrangian reference frame). The particles do not follow the fluid exactly due to their inertia and gravity forces and therefore have their own unique path dependent upon their instantaneous drag and weight forces.

In this investigation, the path of a solid particle is emulated by mounting a two component laser velocimeter (LV) system to a two-dimensional traverse system. The fluid velocity in two components is measured by the LV and then sent to the computer. The instantaneous acceleration that a real particle would undergo in each direction is calculated using the measured fluid velocity and the Stokes transport equation. When, the traverse has reached its new desired velocity (5 ms), the fluid velocity is measured and the sequence repeats. This process is repeated 100 times per second which is significantly faster than the estimated Kolmogorov time scale (Hinze, 1975).

The Reynolds number based on channel half-width and centerline velocity ranges from 1800 to 12,000. Comparisons to existing direct numerical simulations are investigated in order to show that this new technique works. Fluid velocity autocorrelation coefficients are shown for both the Eulerian particle-Lagrangian reference frames and presented in this paper.

Introduction

Particle-laden flows are common in industrial applications and real-world situations such as atmospheric transport of pollutants, combustion processes and spray processing of materials. The focus of this investigation is on a particular category of particle-laden flows where there is a dilute amount of solid particles in a gas flow such as air. For this type of flow, a number of assumptions may be made which simplify the full particle transport equation. Particle-particle interactions are assumed to be negligible and the particles do not modify the surrounding turbulence. The density of a solid particle is significantly greater than the density of the gaseous carrier fluid. Stokes flow around each particle can be assumed when the relative velocity between the particle and the fluid is small. Considering the aforementioned assumptions, the particle transport equation may be reduced to,

\[ \frac{d\bar{V}_p}{dt} = \frac{1}{\tau_p} (\bar{U}_f - \bar{V}_p) - \bar{g} \]

where \( \bar{V}_p \) is the particle velocity, \( \bar{U}_f \) is the fluid velocity, \( \tau_p \) is the particle time constant, and \( \bar{g} \) is the acceleration due to gravity (Hinze, 1975).

The important interaction to consider is the dispersion of the particle by the carrier fluid turbulence. However, the dispersion of particles is difficult to determine because the current models require the knowledge of the fluid velocity autocorrelation in the reference frame of the particle (the particle-Lagrangian reference frame). This quantity has not been measured directly experimentally due to the difficulties involved. Many experimental studies such as Snyder & Lumley (1971) and Wells & Stock (1983) have been performed which measure the particle dispersion. The fluid velocities as seen by the particle cannot be directly measured. Direct numerical simulations (DNS) [Eaton et al, Elgobashi et al], have obtained detailed velocity data in the particle-Lagrangian reference frame and have shed some light on this area. Unfortunately, these studies are constrained to low Reynolds number flows due to the amount of computer power and time required to resolve the increasing range of turbulence scales.

A new technique is presented herein which overcomes many of the limitations encountered in previous studies. This technique combines numerical and experimental methods. The experimental side of the project consists of generating a turbulent flow in a high aspect-ratio, fully-developed water channel flow. The water in the channel is used to instantaneously solve the Navier-Stokes equations at a specified Reynolds number, as would a direct numerical
A control system (detailed in the Control System section) was developed in order to emulate a particle. This technique is a closed loop digital control system where the particle velocity is the output and the fluid velocity (as seen by the particle) is the continually changing input. In one cycle of the control loop the fluid velocity was measured, used as an input to the transport equation, and the resulting velocity was sent to the traverse (emulated particle). The performance of this control cycle is critical. Therefore, the frequency at which the emulated particle responds to the flow must be greater than the highest turbulent frequencies present in the flow.

In this paper, the technique will be presented and discussed. Characterization and qualification of this technique will be presented along with data for flow conditions and particle parameters matching recent studies. In particular, our results will be compared to recent numerical simulations performed at Stanford University.

**Experimental apparatus**

The experimental apparatus consisted of a recirculating water tank with a long, thin rectangular area as the test section. The test section was approximately 5 meters in overall length with the first half of this section used as a development region. The channel half-width was 2 cm and the aspect ratio was 12.5. The water in the tank was driven by a single marine-type propeller which moved the water at a maximum velocity of 0.65 m/s (6.5 kg/s) in the centerline of the test section. Preceding the test section was an entrance region which contained flow straighteners, wire mesh, and a 5th order polynomial contraction. The experimental apparatus, shown in Figure 1, is described in more detail in Beckel, (1994).

A two-directional laser velocimeter (LV) system, manufactured by TSI Corp., was used for this investigation. This system consists of a ColorBurst Multicolor Beam Separator, a ColorLink Multicolor Receiver, and a IFA-750 Digital Burst Correlator. A TSI 4-beam (2 direction) LV probe, connected to the ColorBurst via a fiber optic cable, was used. This probe was mounted on a traverse, the performance of the control system was critical (analogous to grid refinement). The closed loop control cycle can be divided into three main sections; measuring fluid velocities, processing the velocity data, and accelerating the traverse. Each of these control cycle components will be discussed separately in this section.

The first section of the control loop was the measurement of fluid velocities in the water channel. The LV system was used to measure the stream-wise and stream-normal fluid velocity components in the channel test section. The performance of this part of the cycle depended exclusively on the availability of measured velocities (or velocity data rate). In order for the next part of the control cycle to execute, a new valid fluid velocity must first be available. The variability of the data rate was due to several main factors which included the relative velocity between the LV probe and the fluid and the tracer seeding rate. As the relative velocity between the probe and fluid decreased, the data rate generally decreased. This factor was compensated for by adjusting the seeding rate. Experience has shown that a 5 ms period was required to ensure that a new velocity point was available at least 90% of the time.
Once the fluid velocities were acquired, the second section of the control cycle began. In this section, measured velocity data was transferred from the LV system to the control computer and this data was used to calculate the new particle velocity using the particle transport equation. The computer code required to execute these operations was resident on the DSP board (NI NB-DSP2300). The DSP board controlled the handshaking between the digital input/output board (NI NB-DIO-32F) and the LV system. The handshaking allowed the computer to empty and fill the data buffer in the IFA 750 and transfer the data in the buffer to the memory on the DSP board. Once the velocity data was on the DSP board, the new particle velocities were calculated and then sent out to each traverse using the analog output board (NI NB-AO6).

The use of the DSP board was required because the execution time of the section part of the control cycle was considerably less using the DSP board than the main computer’s CPU. None of the main computer system interrupts affected the DSP board execution because the board was dedicated only one particular operation. The time required to transfer the velocity data from the LV system to the computer, to process the data and to send out the calculated information to the traverses was in the microsecond range and therefore was negligible when compared to the other sections.

The third part of the system was the response of the traverse system. Once the traverse received the signal from the computer, a period of time was required for the traverse to accelerate to the new velocity. This is the settling time which is defined as the time required for the traverse to react to a step input and achieve the desired velocity to within 5%. Each time a new particle velocity was calculated, the traverse velocity was updated and this update came in the form of a step input. The traverse settling time was found to be less than 5 ms for both traverse directions.

The entire system throughput time was limited by both the traverse system response time and the availability of new measured fluid velocities, with the computer processing time being negligible. A full cycle required 10 ms to complete, giving the system an update rate of 100 Hz. Velocity data was measured during the first 5 ms of the cycle and the traverse was accelerated during the second 5 ms of the cycle. The control loop is shown below in Figure 2.

![Figure 2: Control Cycle](image)

Before the cycle started (t=0 ms) the data buffer in the LV processor was cleared and left empty until the traverse achieved its desired velocity (t=5 ms). At this point, the LV data buffer was set by the control code to fill. After a period of 5 ms (t=10 ms) the data buffer was read and transferred to the DSP board if available. In this case the data buffer was cleared and the cycle repeated. If no data was in the buffer after the 5 ms period, the data buffer was not cleared and remained ready to accept any new measured fluid velocity point. The traverse was also not accelerated until such time as a new measured fluid velocity was obtained.

A number of checks were built into the computer control code in order to account for any errant velocity readings. If either the velocity magnitude was outside a predetermined range or if no new velocity data points were available, that velocity was discarded and the current traverse velocities remained constant. If no new data point was available during the 5 ms period, the traverse was not accelerated until a new velocity was measured. Rarely was there no new velocity point available for more than 10 ms and the those runs were disregarded if the maximum time between velocity measurements exceeded the kolmogorov time scale. Fluid measurements taken in the Eulerian reference frame are not restricted by the time requirements of the control cycle and therefore the velocity sampling rate may be significantly higher.

**Results**

**Part 1: System Characterization**

In order for this new technique to work, the time for the completion of one control loop cycle had to be less than the smallest turbulent time scale (Kolmogorov time scale) present in the flow by at least a factor of two or three. This ensured that the smallest scales were resolved. As mentioned in the previous section, the control loop consists of three sections, each of which will be discussed separately in this section.

Section 1 of the control cycle consisted of measuring fluid velocities with the laser velocimeter. As mentioned previously, the LV data rate varies depending upon the flow conditions. It was found that a 5 ms period was required to consistently acquire a new valid data point.

Measured velocity data was transferred from the LV system to the computer and the new particle velocity was calculated from the particle transport equation during the second part of the cycle. These operations were performed on the DSP board and their execution was completed in 3 to 4 microseconds. The computer code had minimal overhead since the DSP was dedicated exclusively to executing the code without any system interrupts. The execution time was negligible when compared to the other two sections of the control cycle.

The third section of the cycle was the traverse response to a step input. The steady-state and transient response of the traverse system to a step input was tested by subjecting the traverse to a step function and directly measuring the response of each traverse motor. This response was a function of only the physical characteristics of the traverse system and was independent of the characteristics of rest of the control system. The feedback from the traverse system was monitored by the computer. This feedback indicated the actual velocity
at which the traverse was traveling. The steady-state traverse response to an input voltage was analyzed and was found to vary linearly over a range of input voltages. That is, the traverse velocity varied linearly with a variation of input voltages from the computer. Also, the steady-state error was found to be less than 0.1%.

Each traverse was found to have a settling time of 5 ms when subjected to a step input. The settling time is defined as the time required for the traverse to reach the velocity indicated by the step input to 10% and remain within this error band. Figure 5 below show a typical response curve for each traverse motor normalized by the magnitude of the input step function. The horizontal and vertical traverses had essentially the same response characteristics with a maximum percent overshoot of approximately 10% and a settling time of 5 ms. Achieving this response required careful tuning of the traverse controllers.

\[
\frac{dV_p}{dt} = \dot{V}_p = \frac{U_f - V_p}{\tau_p}
\]

\[
sV_p(s) = \frac{U_f(s) - V_p(s)}{\tau_p}
\]

\[
V_p(s) = \frac{1}{\tau_p s + 1}
\]

To validate this behavior, the traverse (or particle) was initially at rest and was subjected to an approximate step function. The step function was the actual water flow at a constant mean velocity and was approximate because the turbulent flow had a fluctuating component. The traverse was allowed to accelerate from rest as if it were a real particle to the mean fluid velocity using the control cycle.

![Figure 4. Response of Traverse System: ∆ step function; ■ traverse response](image)

Considering the three parts of the system response, the period for one iteration of the control loop was 10 ms (see Figure 4). The first 5 ms of the loop allowed the traverse to reach the desired velocity and the second 5 ms of the loop was the window in which new velocity points were acquired. As mentioned previously, the computational time was on the order of microseconds and therefore can be neglected. Therefore, the update rate for the entire control system was 100 Hz, which was significantly greater than the estimated Kolmogorov time scale of approximately 4 to 10 Hz. A new velocity was not read until the traverse had reached it new desired velocity to prevent any errors in determined the relative velocity between the particle and the fluid. This was critical because the new velocity calculation was dependent upon the computer’s knowledge of the current traverse velocity. Any difference between the actual traverse velocity and the predicted traverse velocity would create an error in calculating the next particle velocity.

A particle responding according to the Stokes transport equation should behave as a first-order system. This can be shown by taking the Laplace transform of the transport equation and rearranging terms in the form of a transfer function as shown below. The mean gravity term was neglected in the following analysis because gravity only affects the particle drift velocity and not its transient response.

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A first-order system subjected to a step function by definition has no overshoot and has a settling time equal to three time constants. Also, the system should reach 63% of the final value of the step input after a time equal to one time system constant. In this case the system time constant was the particle time constant. This means that after one time constant the horizontal traverse velocity should be approximately 63% of the mean flow velocity. In each case, the velocity of the particle was 60-65% of the centerline velocity after a time equal to one time constant. Figure 5 shows this to be the case to within 5% for particle with four different time constants. The time axis for each particle was normalized by the time constant for that particle. The vertical axis is the particle velocity normalized by the mean water velocity. It is important to note that the above cases represent individual runs and not averages. Averages of particle responses due in fact collapse on the ideal case. An individual case are not expected to collapse on the ideal case due to turbulent fluctuations in the flow. Also, occasionally no new measured velocity was obtained and this contributed to a particle lag.

Part 2: Comparison to Existing Work

The channel had a wide range of operating Reynolds numbers, and therefore a variety of previous studies can be compared to. For this paper, only the operating condition
which matched recent work by Rouson and Eaton at Stanford University who performed a direct numerical simulation investigation of particle response in a channel flow will be presented. The channel flow parameters at the Tufts facility were matched to the Reynolds number based on channel half-width and centerline velocity for their simulation was 3300 and the Stokes number based on the Kolmogorov time scale was matched at 8.1. A table of turbulence statistics is shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tufts</th>
<th>Stanford</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re (h, Ucl)</td>
<td>3300</td>
<td>3300</td>
</tr>
<tr>
<td>St(k)</td>
<td>8.1</td>
<td>8.1</td>
</tr>
<tr>
<td>Drift velocity (% Ucl)</td>
<td>20%</td>
<td>5%</td>
</tr>
</tbody>
</table>

Table 1: Channel and Particle Parameters

The Eulerian fluid velocity autocorrelation is shown in Figure 6. The stream-wise direction is denoted by 1 and the stream-normal direction by 3. The autocorrelation coefficients are plotted against time.

Figure 6. Eulerian fluid velocity autocorrelation coefficients: \( \bullet R_{11} \); \( \odot R_{33} \).

Figure 7 shows the fluid velocity autocorrelation coefficients measured in the particle-Lagrangian reference frame. The results obtained in this investigation are plotted against the recent results from the Stanford investigation. The Reynolds and Stokes numbers were matched exactly, however the drift velocity was not matched perfectly. The major obstacle was the excessive amount of noise present in the LV velocity readings at low measured fluid velocities. This accounts for some of the problems in the small time separation region on the autocorrelation plot. At this time an acceptable result was not obtained for a smaller drift velocity.

Figure 7. Fluid velocity autocorrelation coefficients in the particle-Lagrangian reference frame: \( \bullet R_{11} \); \( \odot R_{33} \); \( \odot R_{11} \) (2-D constrained); \( \triangle R_{33} \) (2-D constrained)

Limitations

There are several limitations in this work. First, the particle is constrained to move in two dimensions even with a three-dimensional flow. Figure 7 shows the fluid velocity autocorrelation in the particle-Lagrangian reference frame for particles constrained to move in two dimensions only and for particles allowed to move in three dimensions. This figure shows that the autocorrelation is identical for both cases. The second limitation is that the particle time constant used for cannot be less than roughly 0.2 seconds or the system may become unstable due to occasional large accelerations required. This should not be a major limitation since the smallest Stokes numbers of interest correspond to a particle time constant larger than 0.2 seconds. Another limitation is the ability of the LV system to read small velocities accurately with an adequate data rate. This limitation is currently being evaluated and should be surmounted.

Conclusion

The experimental setup was rigorously tested and characterized and performs to within specified parameters. Data was presented to show the traverse response time is 5 msec for each direction. The total time required to complete one complete loop of the control cycle is 10 msec. This update rate of 100 Hz is sufficient to resolve the smallest scales in the flow. The emulated particle was shown to respond as a first order system which is in agreement with the particle transport equation. The fluid velocity autocorrelation measured in the particle-Lagrangian reference frame matches relatively closely to Stanford’s numerical simulation. Experimental difficulties are believed to be the cause of the discrepancies and are surmountable.

References


